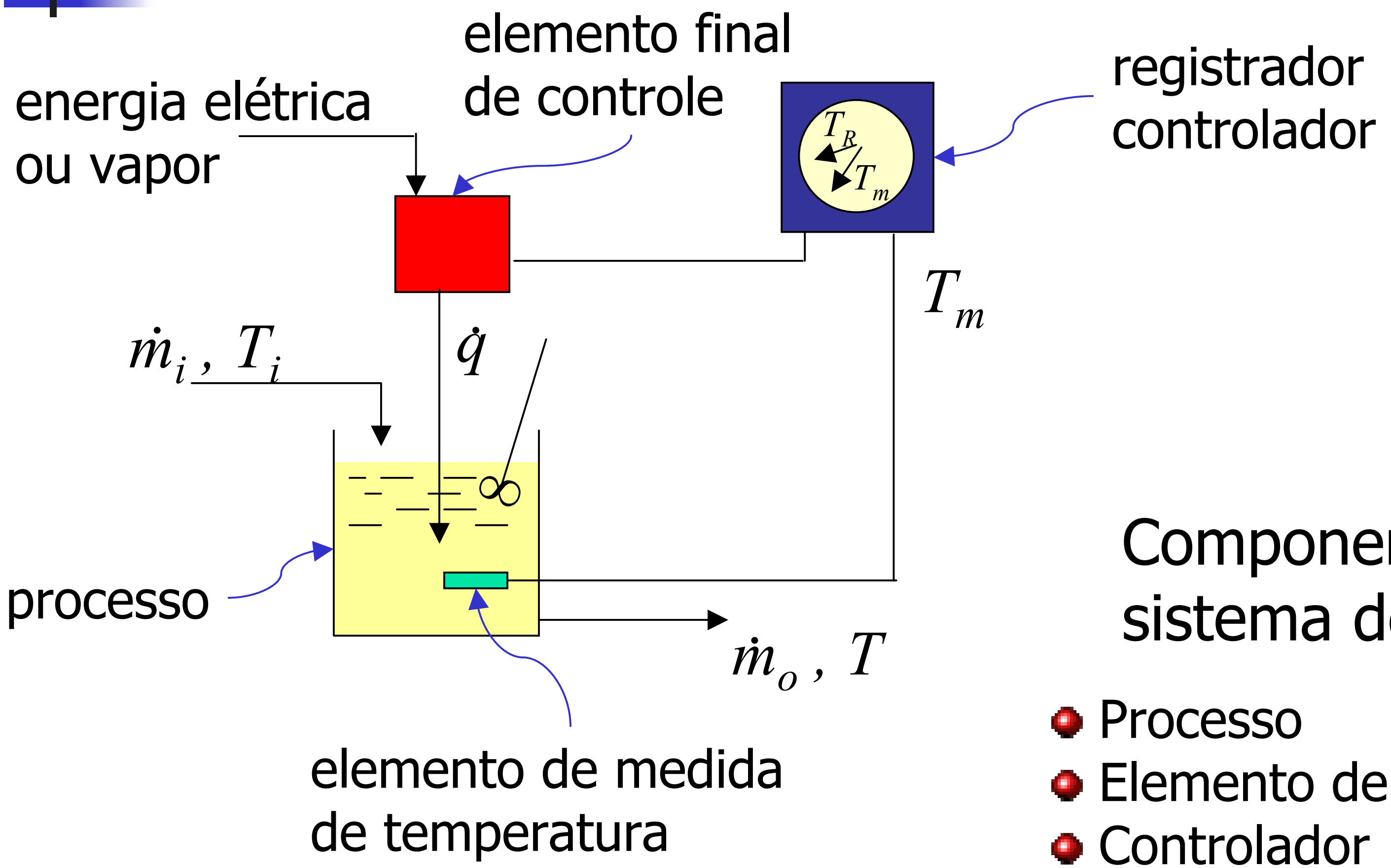
SISTEMAS LINEARES EM MALHA FECHADA

O sistema de controle

Sistema de controle para um tanque de aquecimento

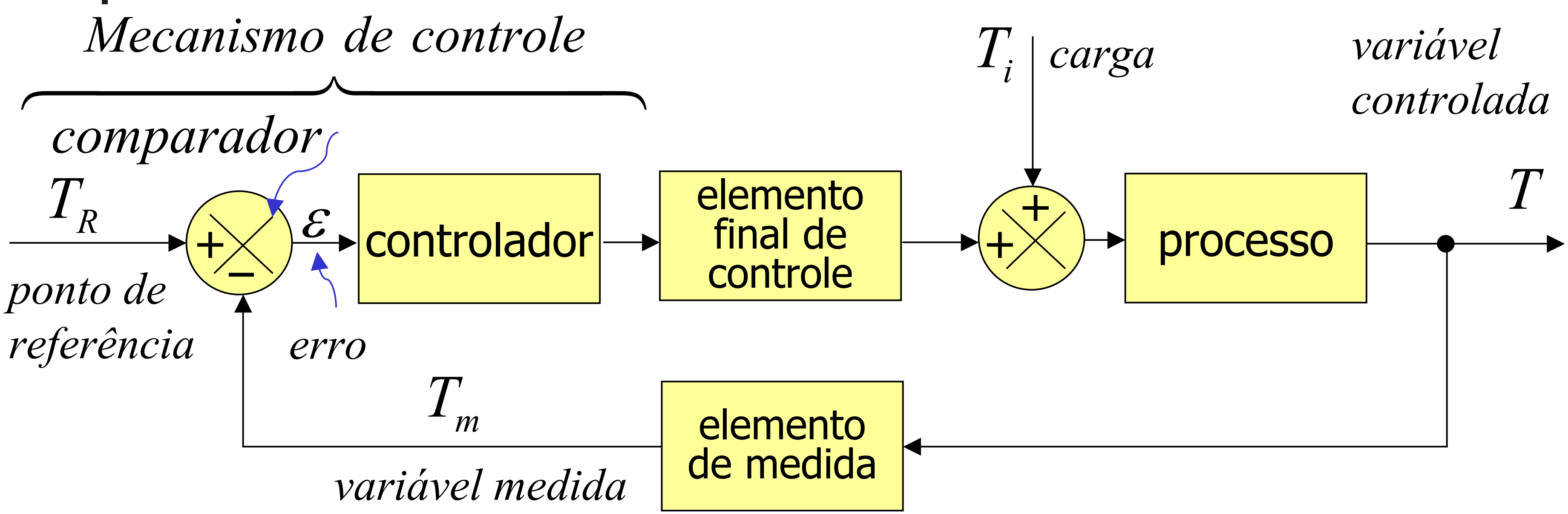


Componentes de um sistema de controle:

- Processo
- Elemento de medida
- Controlador
- Elemento final de controle

Diagrama de Blocos

O diagrama torna muito mais fácil a visualização das relações existentes entre os diversos sinais



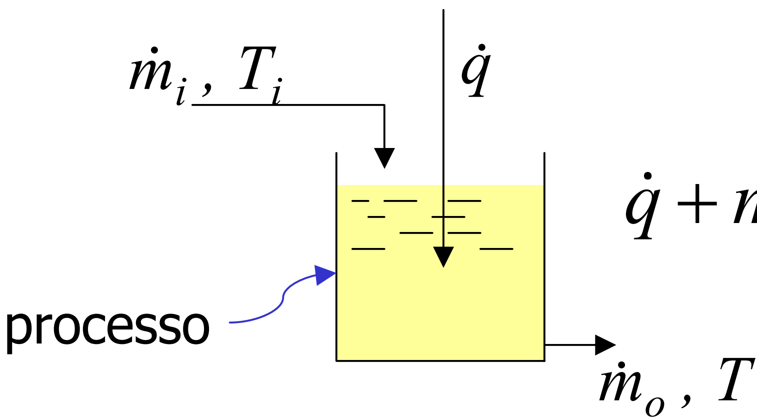
Realimentação negativa *x* Realimentação positiva

Problema servo *x* Problema regulador

Desenvolvimento do Diagrama de Blocos

Processo

Vazão constante $\rightarrow \dot{m}_i = \dot{m}_o = \dot{m}$



Conservação da energia:

$$\dot{q} + \dot{m}C\left(T_i - T_0\right) - \dot{m}C\left(T - T_0\right) = \rho\, C V \frac{dT}{dt}$$

T_0=*Temperatura de referência da entalpia*

Em regime permanente: $\dot{q}_s + \dot{m}C\left(T_{i_s} - T_0\right) - \dot{m}C\left(T_s - T_0\right) = 0$

Subtraindo:

$$\dot{q} - \dot{q}_s + \dot{m}C\left[\left(T_i - T_{i_s}\right) - \left(T - T_s\right)\right] = \rho\, C V \frac{d\left(T - T_s\right)}{dt}$$

VARIÁVEIS DESVIO

$$\hat{T}_i = T_i - T_{i_s} \qquad \hat{T} = T - T_s \qquad \dot{Q} = \dot{q} - \dot{q}_s$$

$$\dot{q} - \dot{q}_s + \dot{m}C\left[\left(T_i - T_{i_s}\right) - \left(T - T_s\right)\right] = \rho\, C V \frac{d\left(T - T_s\right)}{dt}$$

$$\dot{Q} + \dot{m}C\left(\hat{T}_i - \hat{T}\right) = \rho\, C V \frac{d\,\hat{T}}{dt} \quad \Rightarrow \quad \frac{\dot{Q}}{\dot{m}C} + \hat{T}_i - \hat{T} = \frac{\rho V}{\dot{m}} \frac{d\,\hat{T}}{dt}$$

Aplicando a transformada de Laplace com $\tau = \dfrac{\rho V}{\dot{m}}$

$$\frac{\dot{Q}(s)}{\dot{m}C} + \hat{T}_i(s) - \hat{T}(s) = \tau\, s \hat{T}(s)$$

$$\hat{T}(s)(\tau\, s + 1) = \frac{\dot{Q}(s)}{\dot{m}C} + \hat{T}_i(s)$$

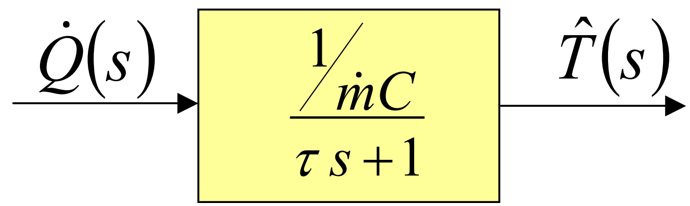
$$\hat{T}(s)(\tau s+1)=\frac{\dot{Q}(s)}{\dot{m}C}+\hat{T}_i(s)$$

$$\hat{T}(s)=\frac{1/\dot{m}C}{\tau s+1}\dot{Q}(s)+\frac{1}{\tau s+1}\hat{T}_i(s)$$

Se houver variação apenas em $\dot{Q}(s)$, então $\hat{T}_i(s)=0$

e a função de transferência que relaciona $\hat{T}(s)$ a $\dot{Q}(s)$ é:

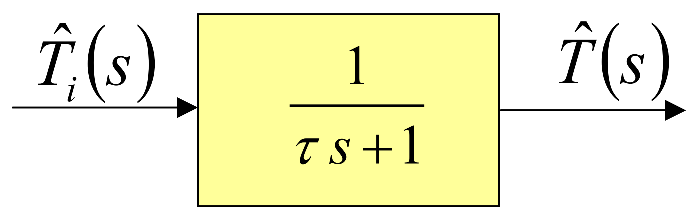
$$\frac{\hat{T}(s)}{\dot{Q}(s)}=\frac{1/\dot{m}C}{\tau s+1}$$



Se houver variação apenas em $\hat{T}_i(s)$, então $\dot{Q}(s)=0$

e a função de transferência que relaciona $\hat{T}(s)$ a $\hat{T}_i(s)$ é:

$$\frac{\hat{T}(s)}{\hat{T}_i(s)}=\frac{1}{\tau s+1}$$



$$\hat{T}(s) = \frac{1/\dot{m}C}{\tau s + 1}\,\dot{Q}(s) + \frac{1}{\tau s + 1}\hat{T}_i(s)$$

Esta equação é representada no diagrama de blocos

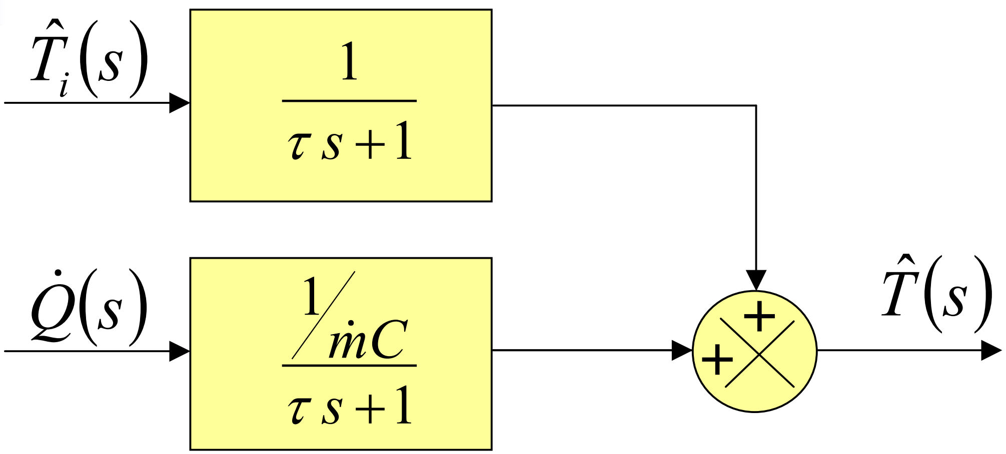
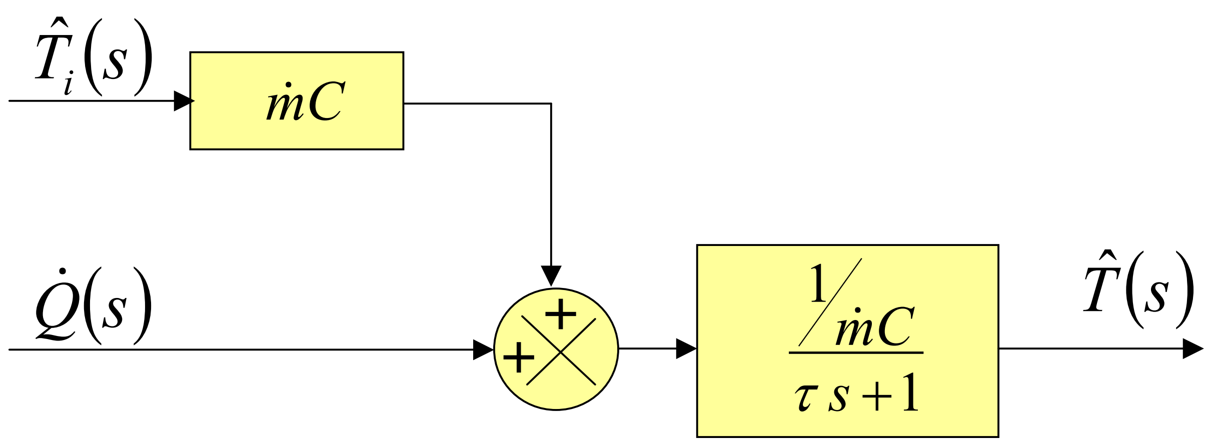
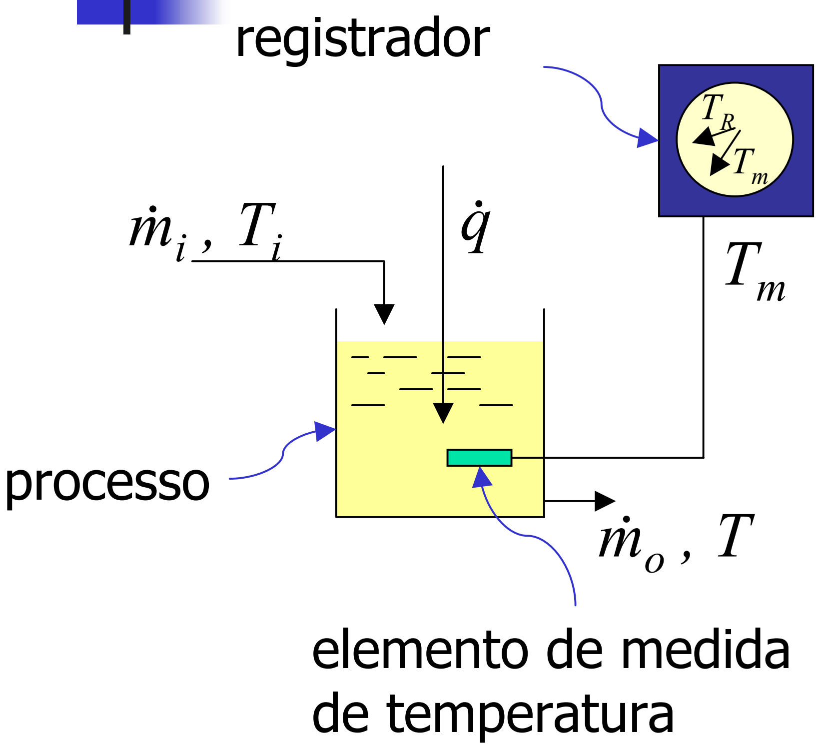


Diagrama de blocos equivalente



Elemento de Medida

O elemento de medida de temperatura pode apresentar algum retardo por transporte



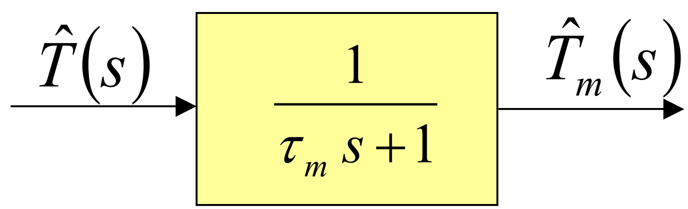
$$\frac{\hat{T}_m(s)}{\hat{T}(s)} = \frac{1}{\tau_m\, s + 1}$$

onde:

$\tau_m =$ constante de tempo do termômetro

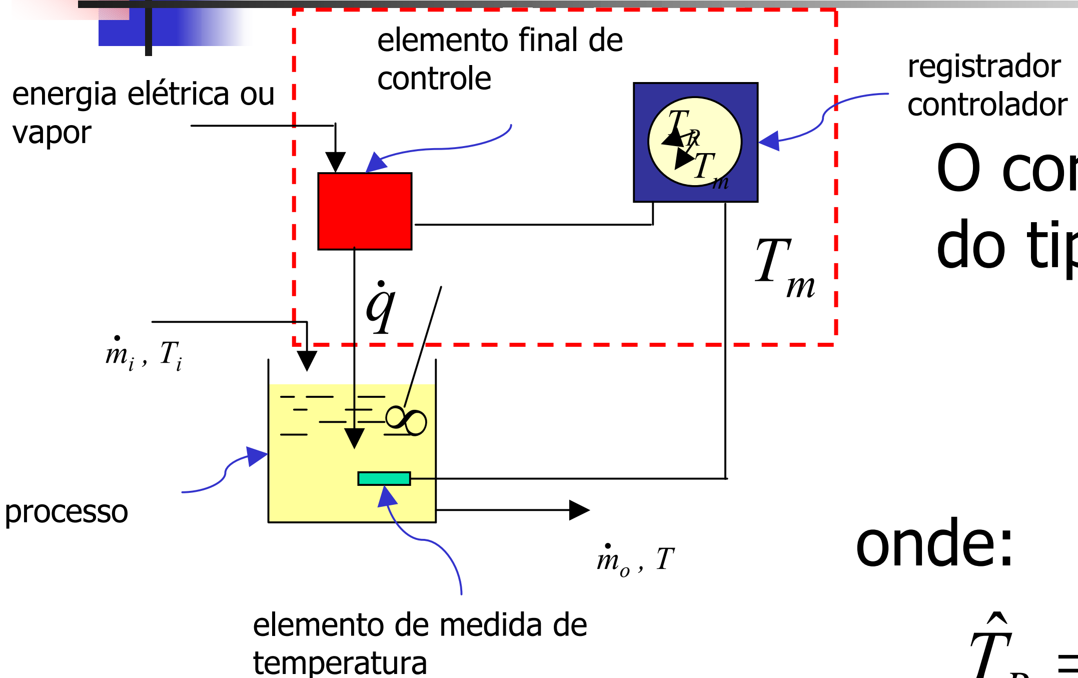
$$\hat{T}_m = T_m - T_{m_s}$$

$$\hat{T} = T - T_s$$



Controlador e Elemento Final de Controle

Por conveniência, os blocos que representam o controlador e o elemento final de controle são combinados em um só bloco



O controlador será considerado do tipo PROPORCIONAL

$$\dot{q} = K_c \varepsilon + A$$

onde: $$\varepsilon = \hat{T}_R - \hat{T}_m$$

$$\hat{T}_R = T_R - T_{R_s}$$

$\hat{T}_{R_s}$ = temperatura de referência "normal"

K_c = sensibilidade proporcional ou ganho do controlador

A = fluxo de calor quando $\varepsilon = 0$

O uso da variável $\hat{T}_R$ permite-nos considerar os efeitos das variações no ponto de referência.

O sistema deve ser projetado para que $T_{R_s} = T_R$

Em regime permanente $\varepsilon = 0$, pois: $\hat{T}_R = \hat{T}_m = \hat{T}$

$$\dot{q} = K_c \varepsilon + A$$

$$\dot{q}_s = K_c 0 + A$$

$$\Rightarrow$$

$$\dot{q} - \dot{q}_s = K_c \varepsilon$$

$$\dot{Q} = K_c \varepsilon$$

Transformada de Laplace: $\dot{Q}(s) = K_c \varepsilon(s)$

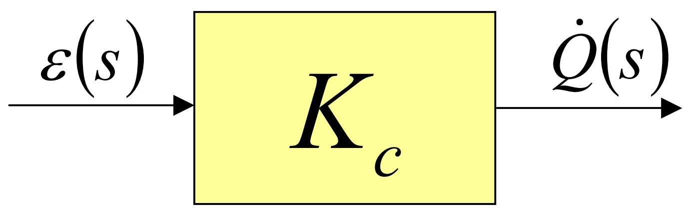
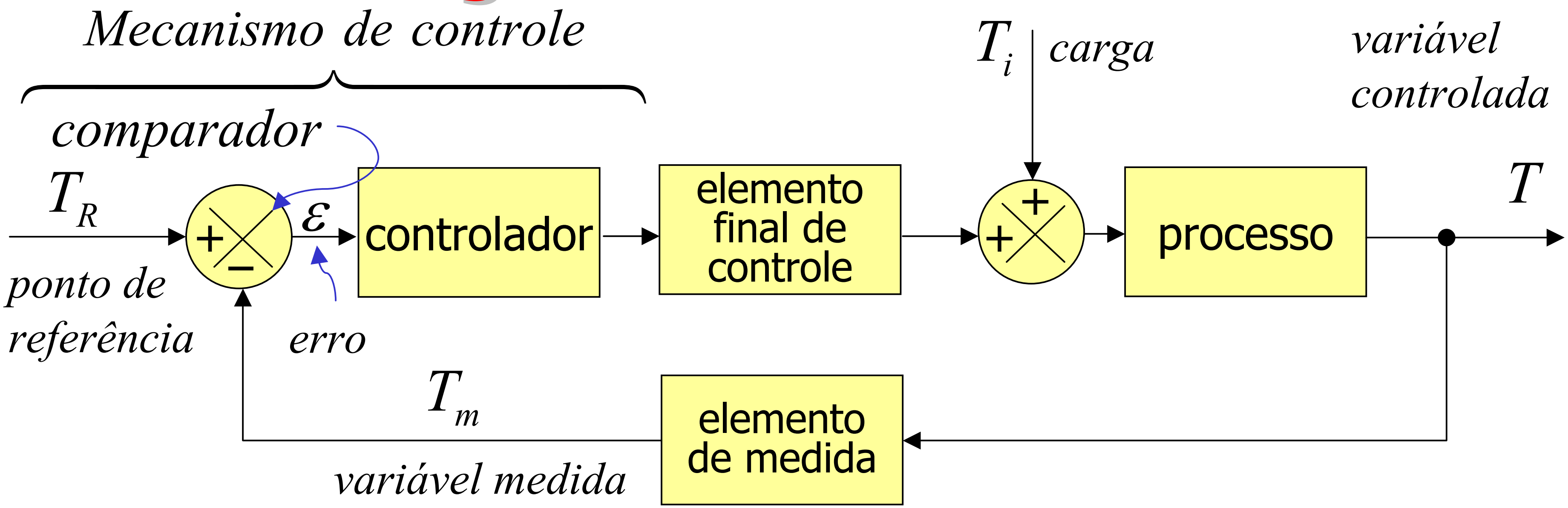
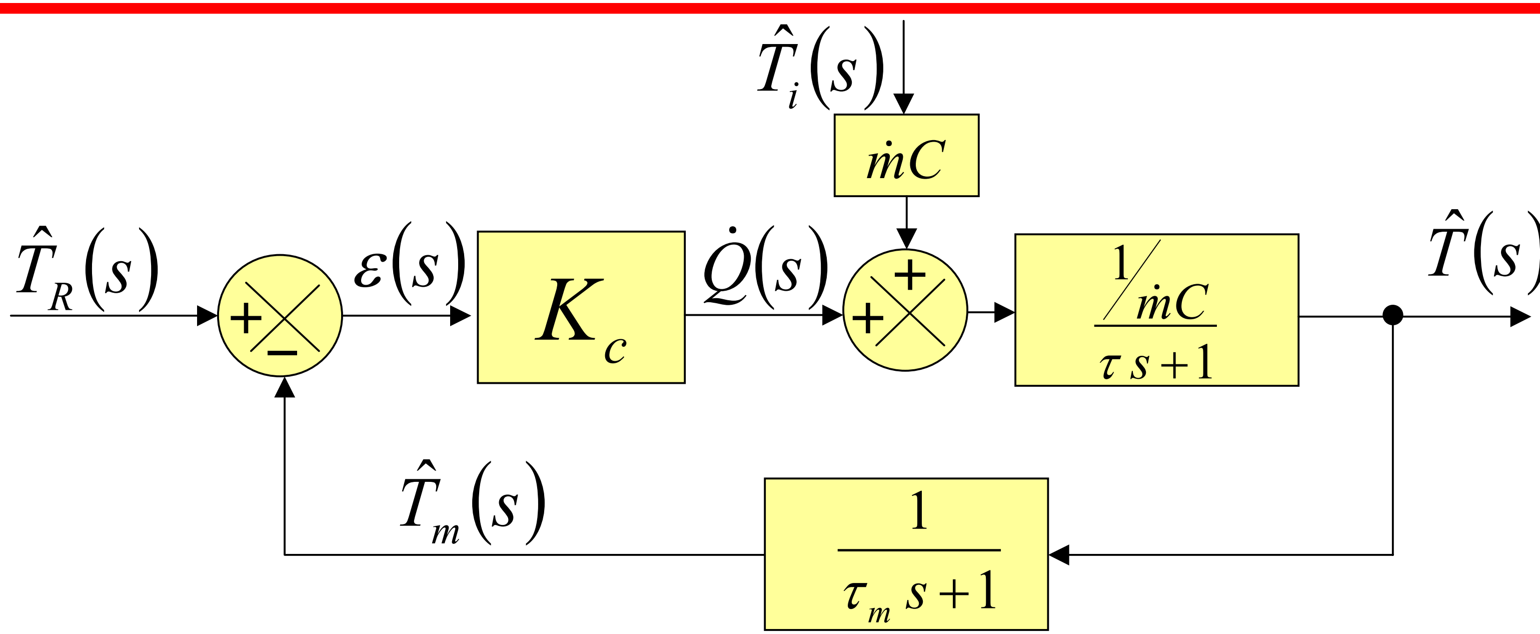
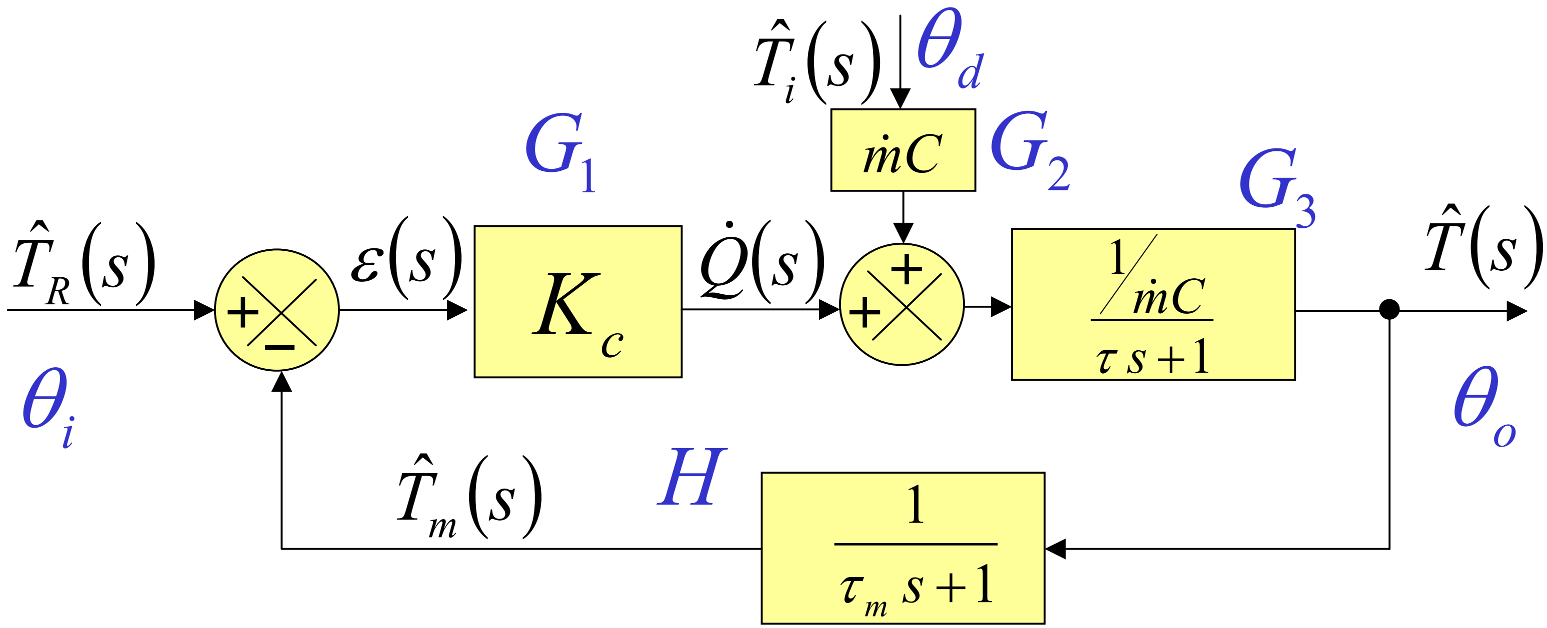


Diagrama de Blocos





Função de transferência do sistema



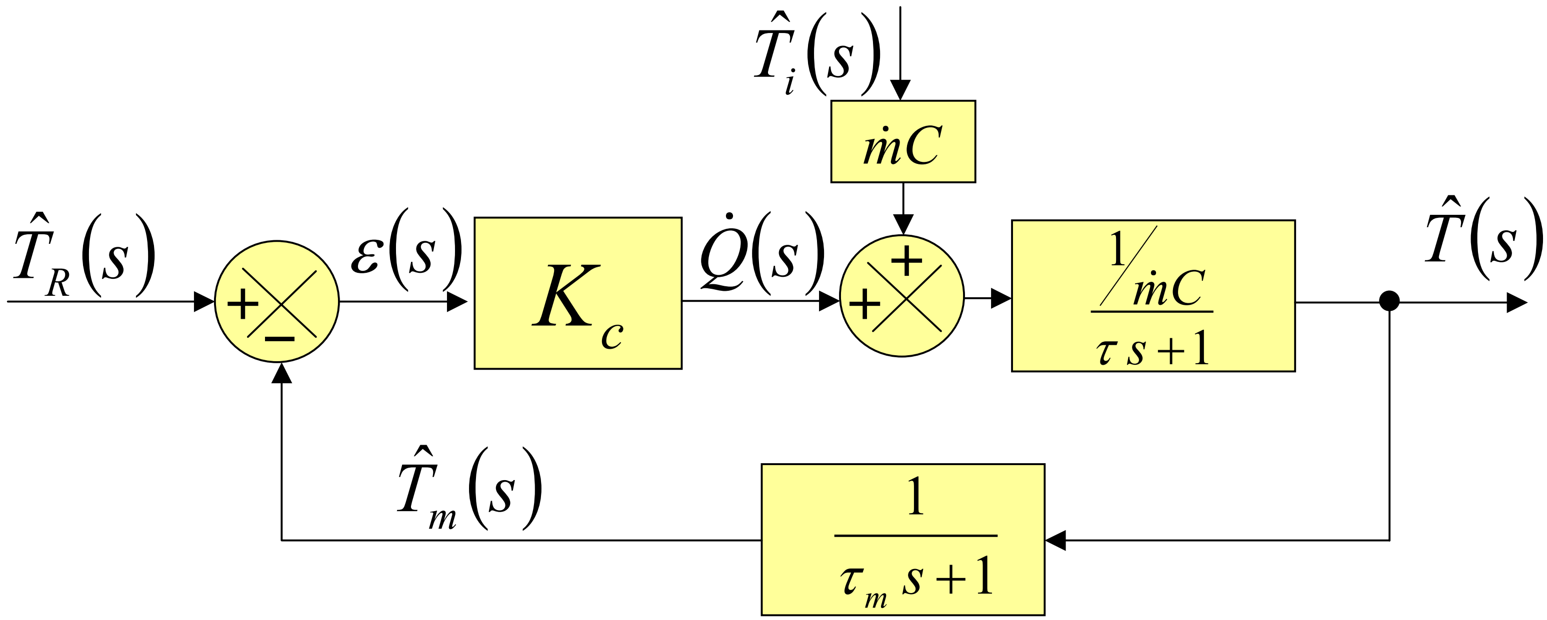
$$\left[\left(\theta_i - H\theta_o\right)G_1 + \theta_d\, G_2\right]G_3 = \theta_o \qquad \theta_o = \frac{G_1 G_3}{1 + HG_1G_3}\theta_i + \frac{G_2G_3}{1+HG_1G_3}\theta_d$$

$$\hat{T}(s) = \frac{K_c \dfrac{1/\dot{m}C}{\tau s+1}}{1 + \dfrac{1}{\tau_m s+1} K_c \dfrac{1/\dot{m}C}{\tau s+1}}\hat{T}_R(s) + \frac{\dot{m}C \dfrac{1/\dot{m}C}{\tau s+1}}{1 + \dfrac{1}{\tau_m s+1} K_c \dfrac{1/\dot{m}C}{\tau s+1}}\hat{T}_i(s)$$

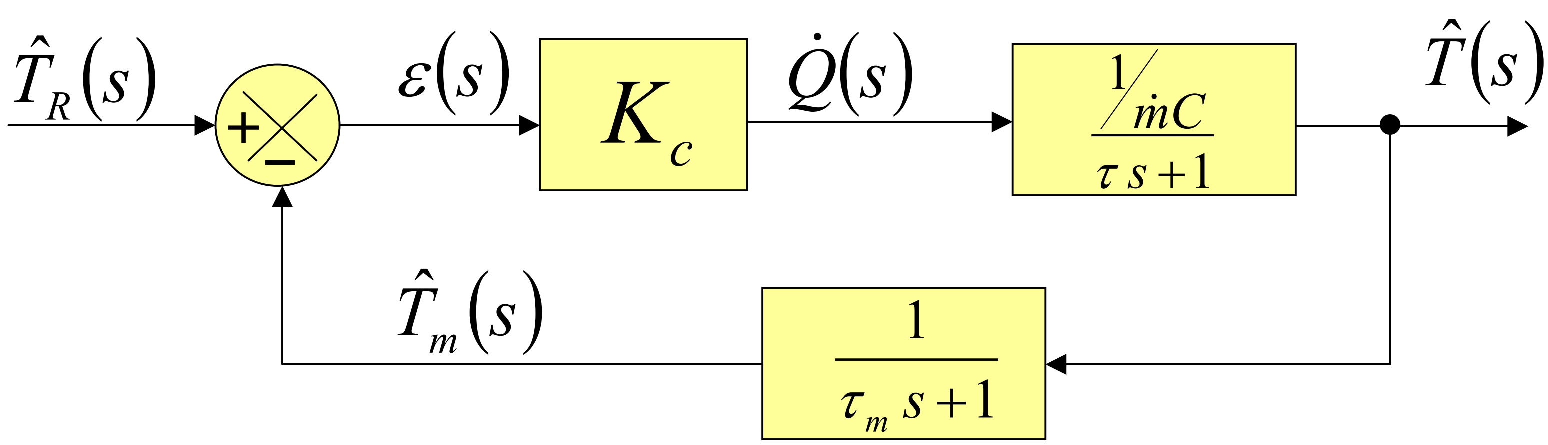
$$\hat{T}(s) = \frac{K_c \frac{1/\dot{m}C}{\tau s + 1}}{1 + \frac{1}{\tau_m s + 1} K_c \frac{1/\dot{m}C}{\tau s + 1}} \hat{T}_R(s) + \frac{\dot{m}C \frac{1/\dot{m}C}{\tau s + 1}}{1 + \frac{1}{\tau_m s + 1} K_c \frac{1/\dot{m}C}{\tau s + 1}} \hat{T}_i(s)$$

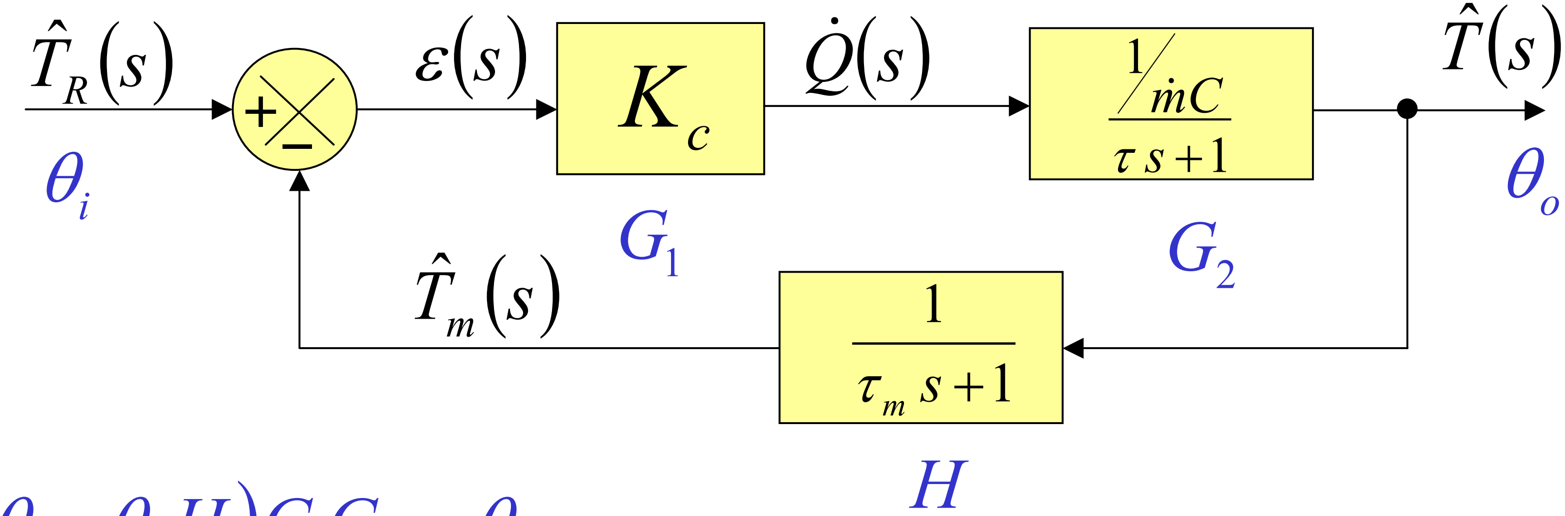
$$\hat{T}(s) = \frac{\frac{K_c}{\dot{m}C}(\tau_m s + 1)}{(\tau_m s + 1)(\tau s + 1) + \frac{K_c}{\dot{m}C}} \hat{T}_R(s) + \frac{(\tau_m s + 1)}{(\tau_m s + 1)(\tau s + 1) + \frac{K_c}{\dot{m}C}} \hat{T}_i(s)$$

Função de transferência do sistema



Para uma variação em $\hat{T}_R(s)$ e $\hat{T}_i(s)=0$





$$(\theta_i - \theta_o H) G_1 G_2 = \theta_o$$

$$\frac{\theta_o}{\theta_i} = \frac{G_1 G_2}{1 + G_1 G_2 H}$$

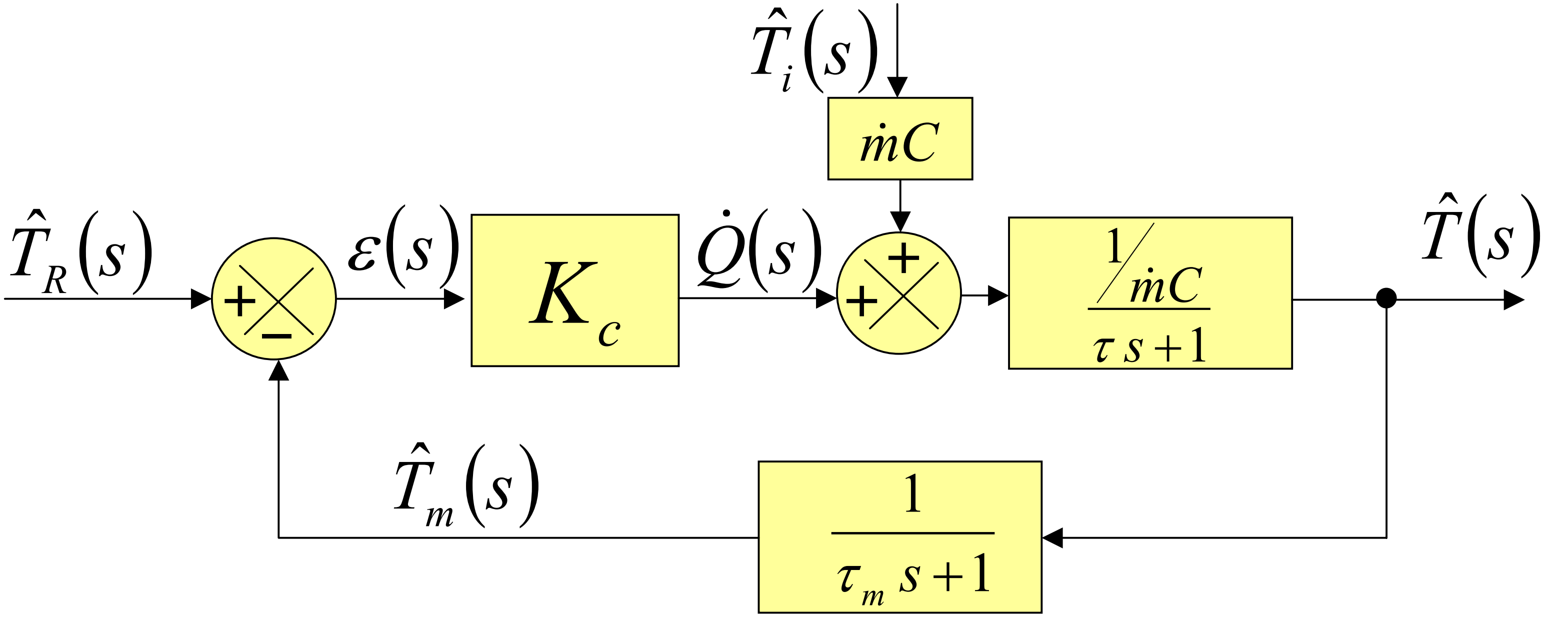
$$\frac{\hat{T}(s)}{\hat{T}_R(s)} = \frac{K_c \cdot \frac{1/\dot{m}C}{\tau s + 1}}{1 + K_c \cdot \frac{1/\dot{m}C}{\tau s + 1} \cdot \frac{1}{\tau_m s + 1}}$$

$$\frac{\hat{T}(s)}{\hat{T}_R(s)} = \frac{\frac{K_c}{\dot{m}C}(\tau_m s + 1)}{(\tau s + 1)(\tau_m s + 1) + \frac{K_c}{\dot{m}C}}$$

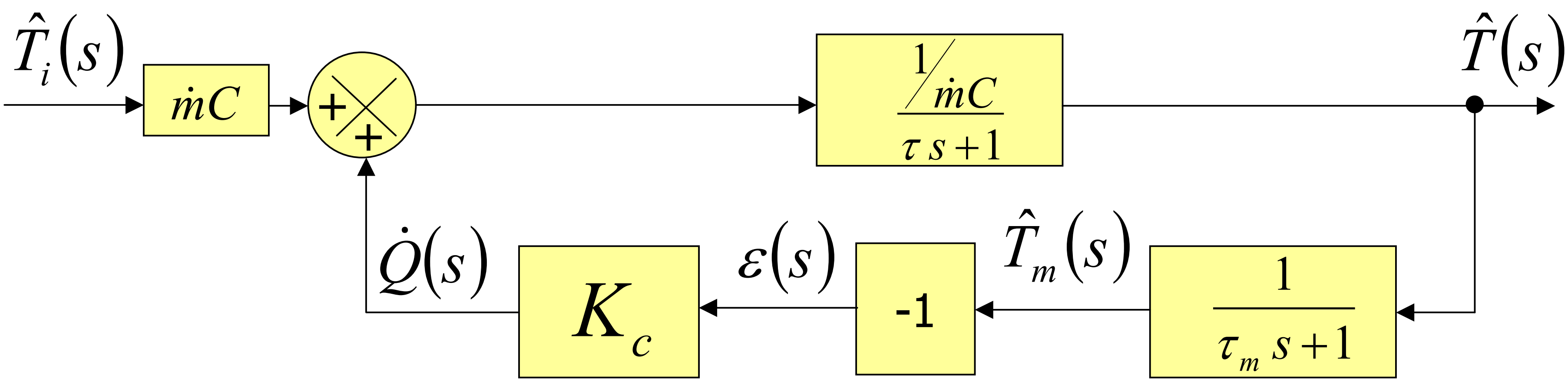
$$\frac{\hat{T}(s)}{\hat{T}_R(s)} = \frac{\frac{K_c}{\dot{m}C}(\tau_m s + 1)}{(\tau s + 1)(\tau_m s + 1) + \frac{K_c}{\dot{m}C}}$$

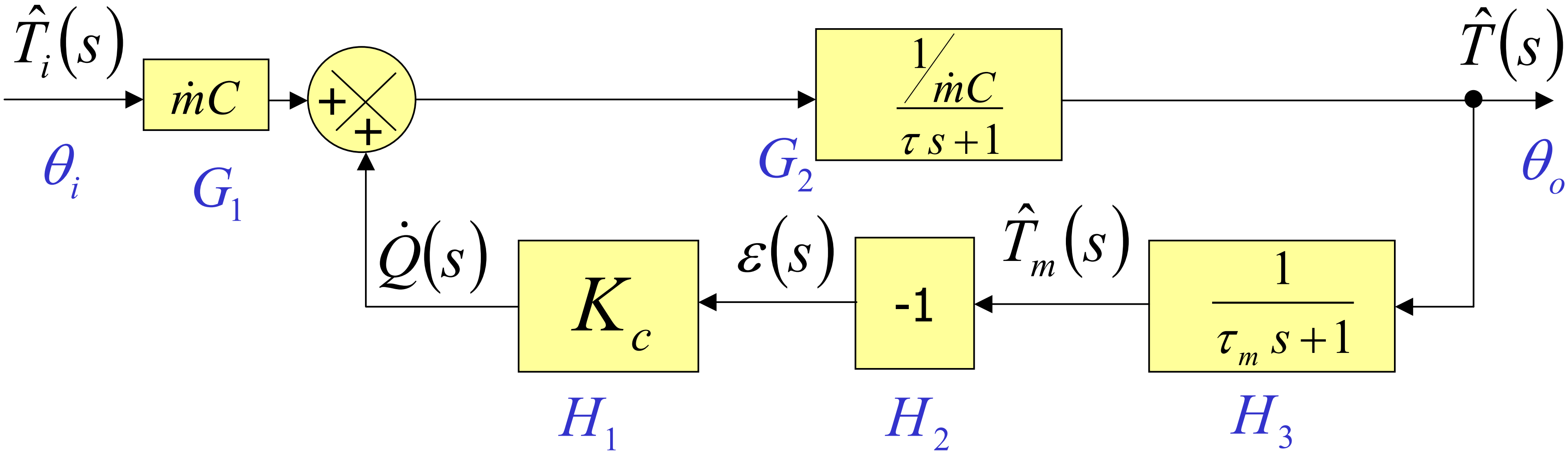
$$\frac{\hat{T}(s)}{\hat{T}_R(s)} = \frac{\frac{K_c}{\dot{m}C}(\tau_m s + 1)}{\tau \tau_m s^2 + (\tau + \tau_m) s + \left(\frac{K_c}{\dot{m}C} + 1\right)}$$

Função de transferência do sistema



Para uma variação em $\hat{T}_i(s)$ e $\hat{T}_R(s)=0$





$$\left(\theta_i G_1 + \theta_o H_1 H_2 H_3\right) G_2 = \theta_o$$

$$\frac{\theta_o}{\theta_i} = \frac{G_1 G_2}{1 - G_2 H_1 H_2 H_3}$$

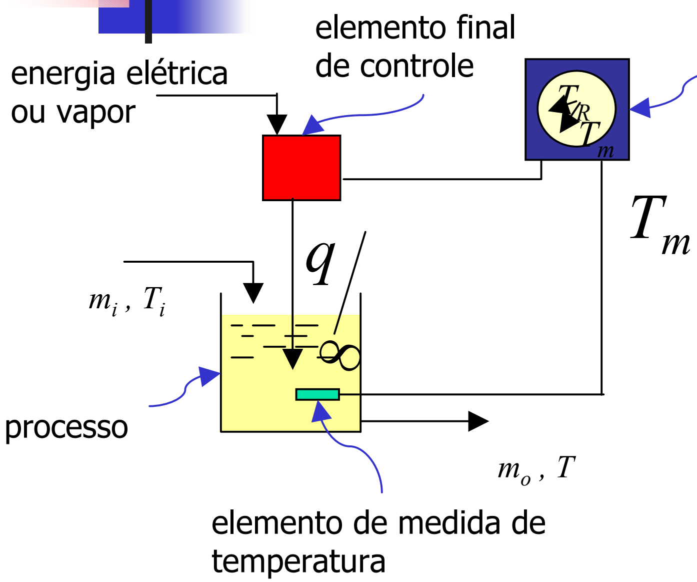
$$\frac{\hat{T}(s)}{\hat{T}_i(s)} = \frac{\dfrac{1}{\tau s+1}}{1 + \dfrac{1/\dot{m}C}{\tau s+1}.K_c.\dfrac{1}{\tau_m s+1}}$$

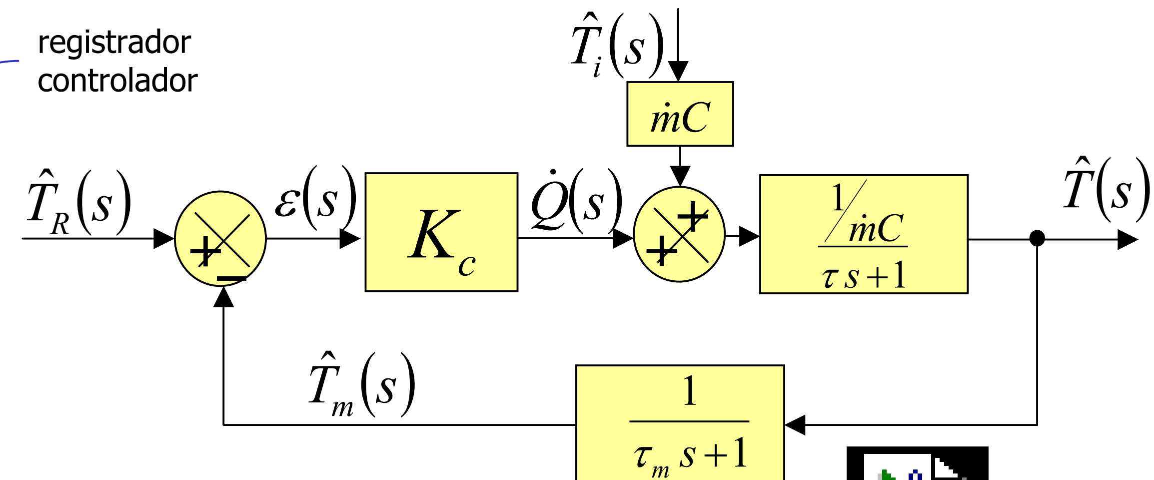
$$\frac{\hat{T}(s)}{\hat{T}_i(s)} = \frac{\dfrac{1}{\tau s + 1}}{1 + \dfrac{1/\dot{m}C}{\tau s + 1} . K_c . \dfrac{1}{\tau_m s + 1}}$$

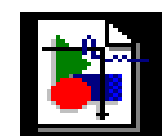
$$\frac{\hat{T}(s)}{\hat{T}_i(s)} = \frac{\tau_m s + 1}{(\tau s + 1)(\tau_m s + 1) + \dfrac{K_c}{\dot{m}C}}$$

$$\frac{\hat{T}(s)}{\hat{T}_i(s)} = \frac{\tau_m s + 1}{\tau \tau_m s^2 + (\tau + \tau_m) s + \left(\dfrac{K_c}{\dot{m}C} + 1\right)}$$

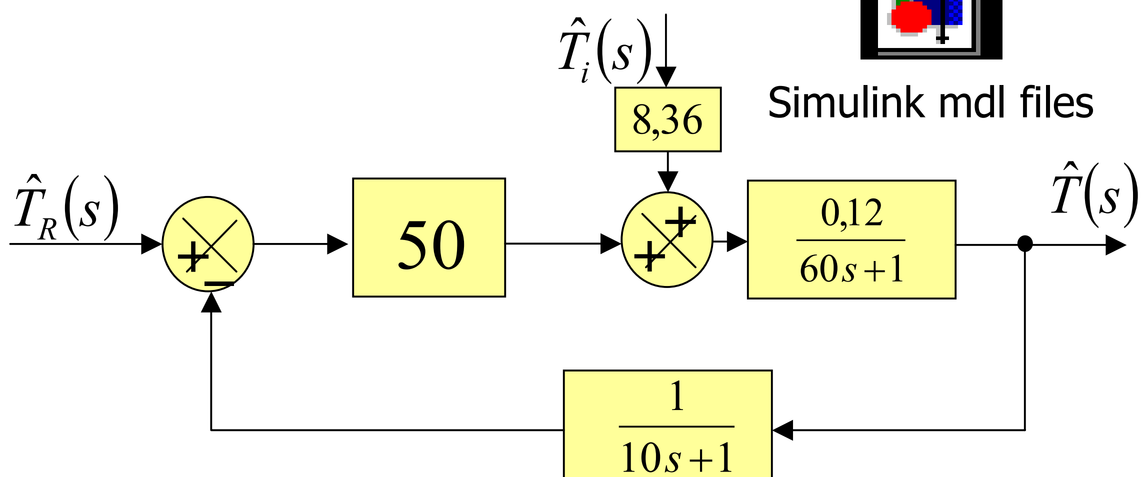
Exemplo de solução com MATLAB Simulink







Simulink mdl files



Dados:

$$\tau = 60\,s$$

$$\tau_m = 10\,s$$

$$\dot{m} = 2{,}0\,{}^{kg}/_{s}$$

$$C = 4{,}18\,{}^{kJ}/_{kg.^oC}$$

$$K_c = 50\,{}^{kW}/_{^oC}$$

Para uma entrada Degrau unitário em T_R, qual o erro em regime permanente?

- Função de transferência:

- Entrada degrau unitário em T_R:

$$\frac{\hat{T}(s)}{\hat{T}_R(s)} = \frac{\frac{K_c}{\dot{m}C}(\tau_m s + 1)}{\tau \tau_m s^2 + (\tau + \tau_m)s + \left(\frac{K_c}{\dot{m}C} + 1\right)}$$

$$\hat{T}_R(s) = \frac{1}{s}$$

$$\hat{T}(s) = \frac{\frac{K_c}{\dot{m}C}(\tau_m s + 1)}{\tau \tau_m s^2 + (\tau + \tau_m)s + \left(\frac{K_c}{\dot{m}C} + 1\right)} \frac{1}{s}$$

- ✓ Teorema do valor final:

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$$

- Valor final: $\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

$$\lim_{t\to\infty} T(t) = \lim_{s\to 0} s.\frac{\frac{K_c}{\dot{m}C}(\tau_m s+1)}{\tau\tau_m s^2+(\tau+\tau_m)s+\left(\frac{K_c}{\dot{m}C}+1\right)}\frac{1}{s} \Rightarrow \lim_{t\to\infty} T(t) = \frac{\frac{K_c}{\dot{m}C}}{\frac{K_c}{\dot{m}C}+1}$$

- Erro: $Erro = T_R(\infty) - T(\infty)$

$$Erro = 1 - \frac{\frac{K_c}{\dot{m}C}}{\frac{K_c}{\dot{m}C}+1} \Rightarrow Erro = \frac{1}{\frac{K_c}{\dot{m}C}+1}$$

Para uma entrada Degrau unitário em T_i, qual o erro em regime permanente, em relação à T_R?

- Função de transferência:

$$\frac{\hat{T}(s)}{\hat{T}_i(s)} = \frac{\tau_m s + 1}{\tau \tau_m s^2 + (\tau + \tau_m) s + \left(\frac{K_c}{\dot{m}C} + 1\right)}$$

- Entrada degrau unitário em T_i:

$$\hat{T}_i(s) = \frac{1}{s}$$

$$\hat{T}(s) = \frac{\tau_m s + 1}{\tau \tau_m s^2 + (\tau + \tau_m) s + \left(\frac{K_c}{\dot{m}C} + 1\right)} \frac{1}{s}$$

✓ Teorema do valor final:

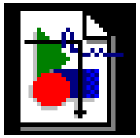
$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$$

- Valor final: $$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

$$\lim_{t\to\infty} T(t) = \lim_{s\to 0} s.\frac{\tau_m s + 1}{\tau\,\tau_m s^2 + (\tau + \tau_m)s + \left(\frac{K_c}{\dot{m}C} + 1\right)}\frac{1}{s} \Rightarrow \lim_{t\to\infty} T(t) = \frac{1}{\frac{K_c}{\dot{m}C} + 1}$$

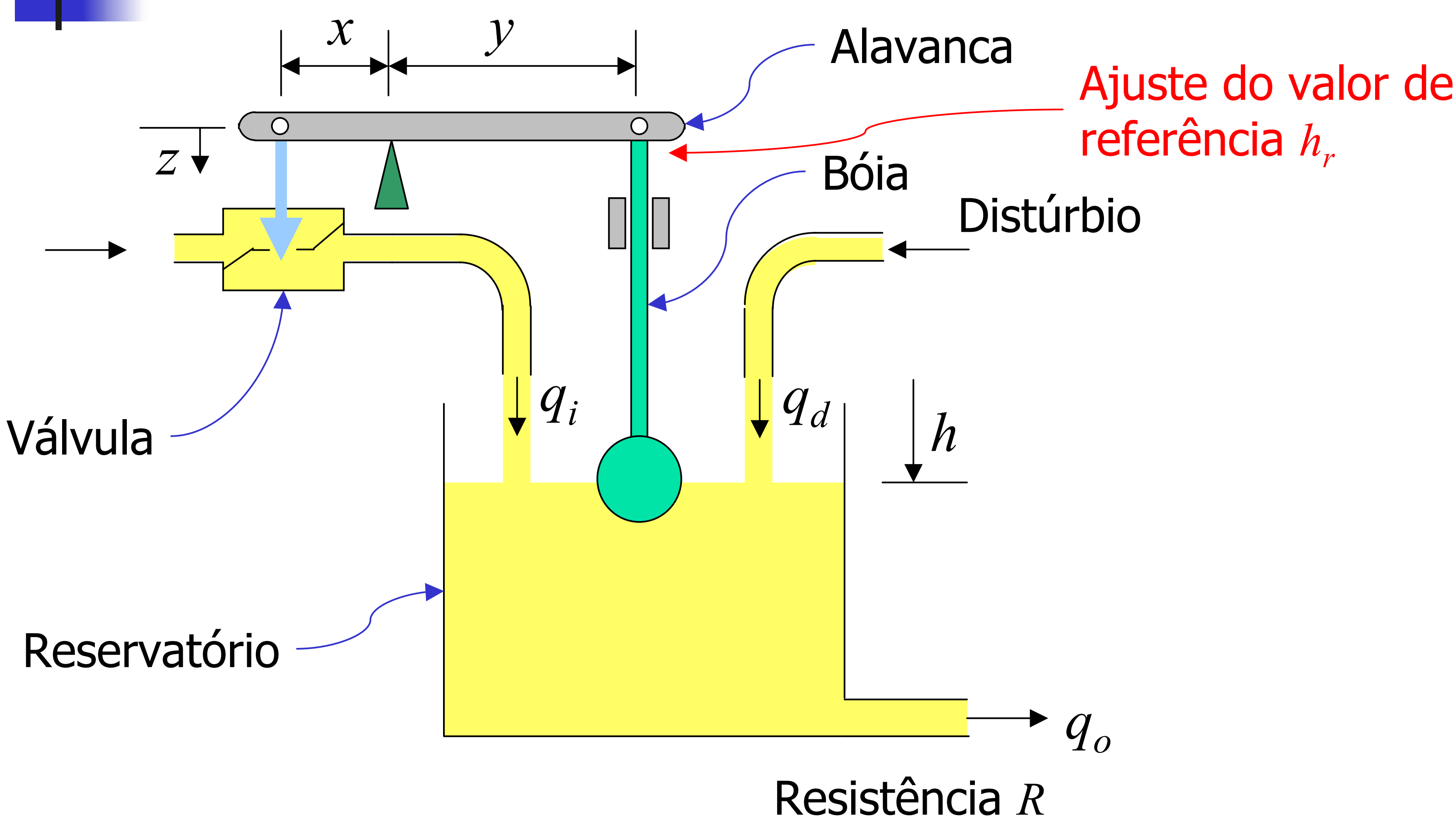
- Erro: $$Erro = T_R(\infty) - T(\infty)$$

$$Erro = 0 - \frac{1}{\frac{K_c}{\dot{m}C} + 1} \Rightarrow Erro = \frac{-1}{\frac{K_c}{\dot{m}C} + 1}$$



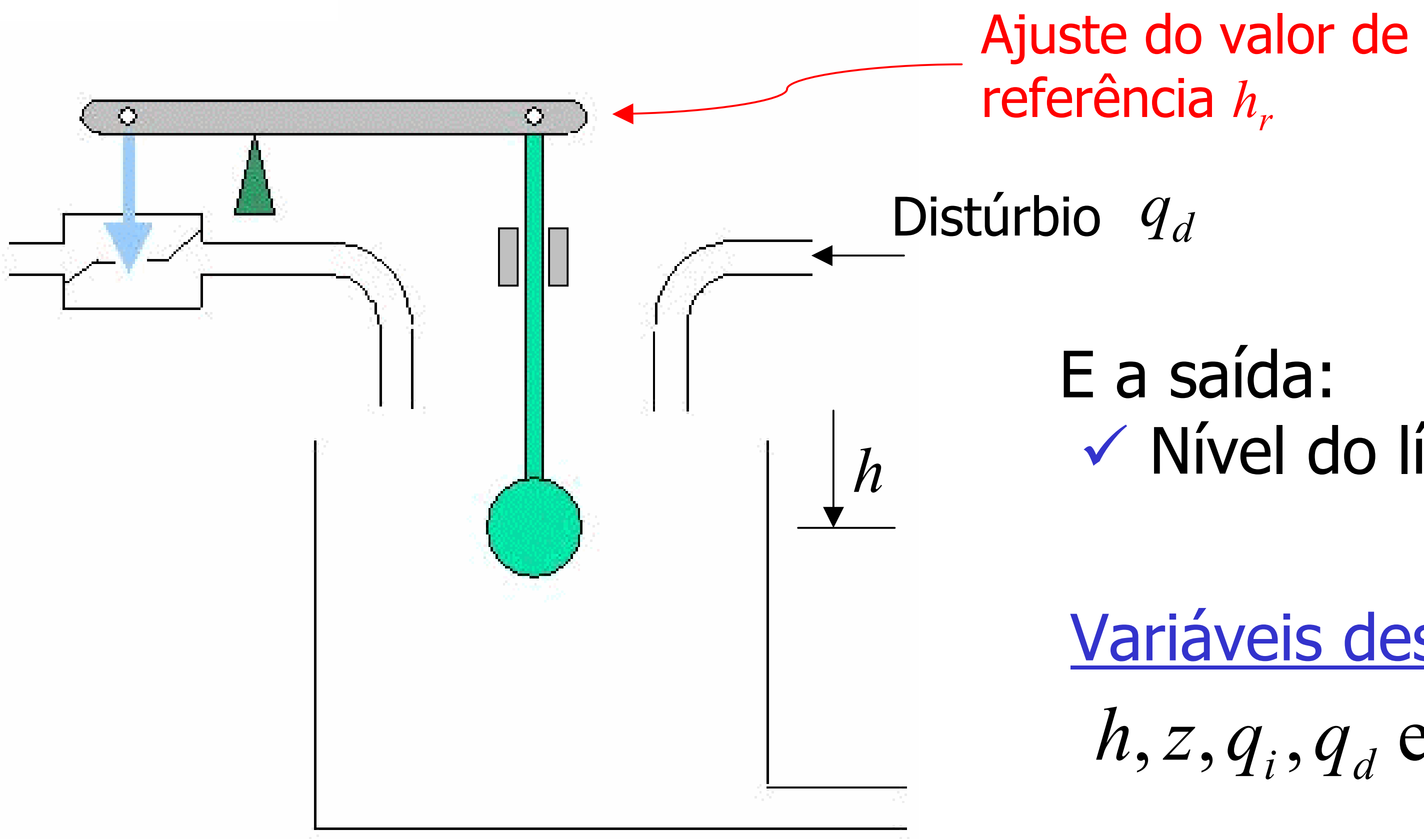
Simulink mdl files

Sistema de controle de nível



Determinar a relação entre as entradas:

- ✓Valor de ajuste para o nível do líquido h_r
- ✓Distúrbio para o nível de líquido q_d



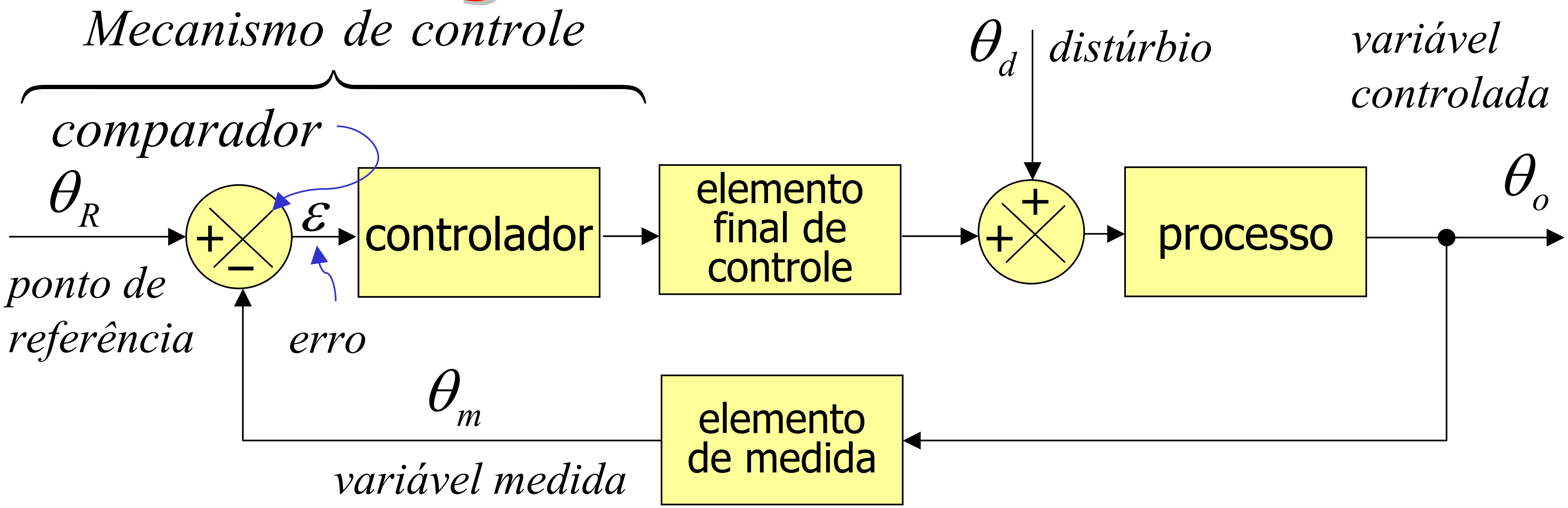
E a saída:

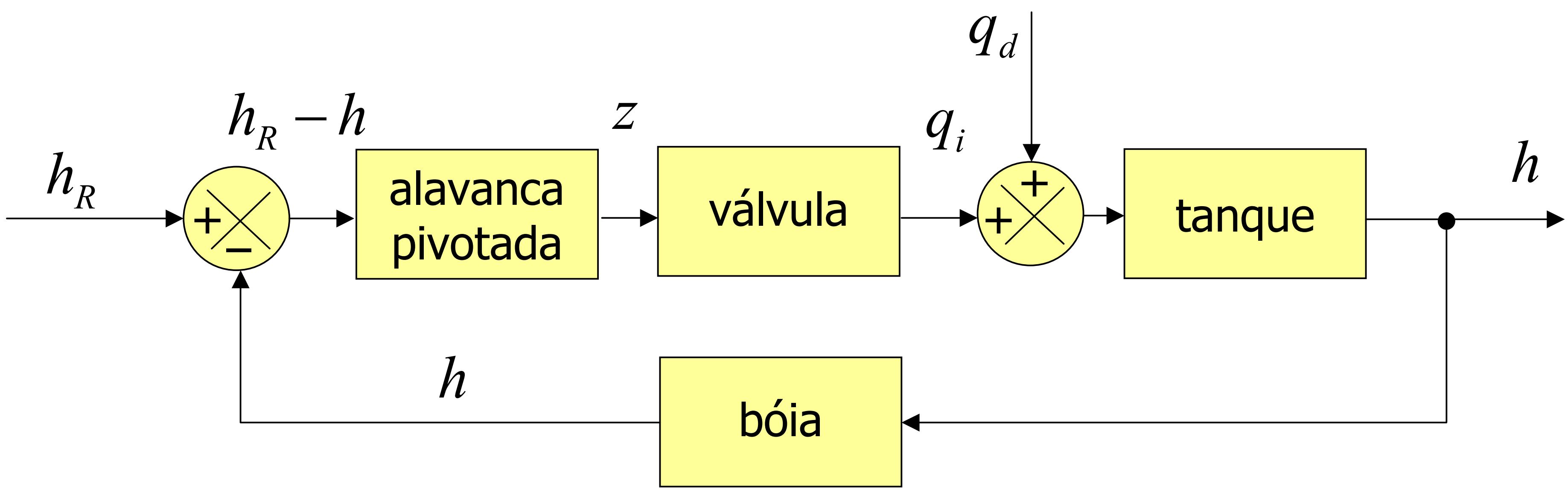
- ✓ Nível do líquido h

Variáveis desvio:

$$h, z, q_i, q_d \text{ e } q_o$$

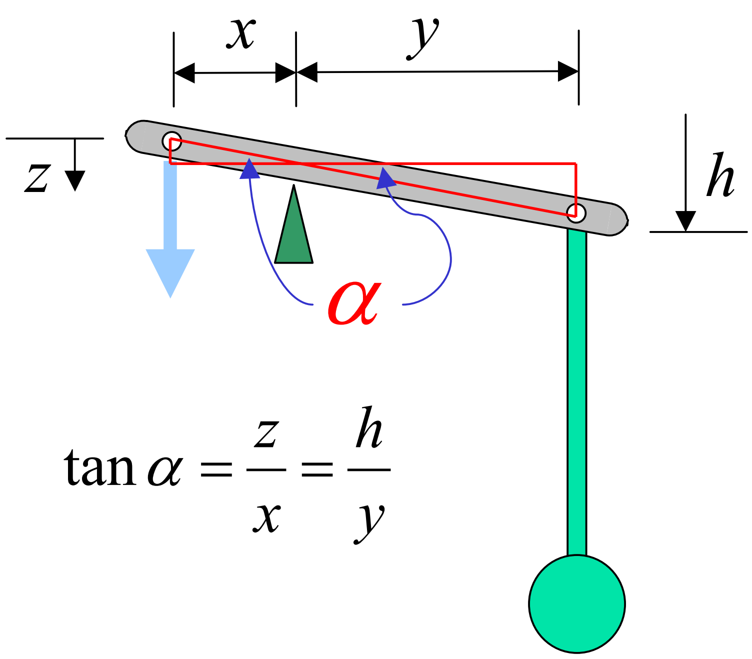
Diagrama de Blocos





Desenvolvimento do Diagrama de Blocos

Alavanca pivotada

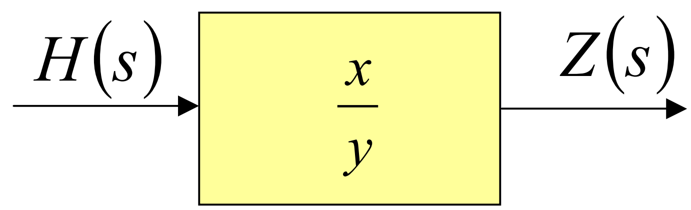


$$\frac{Saída}{Entrada} = \frac{Distância\ do\ pivô\ na\ saída}{Distância\ do\ pivô\ na\ entrada}$$

$$\tan\alpha = \frac{z}{x} = \frac{h}{y}$$

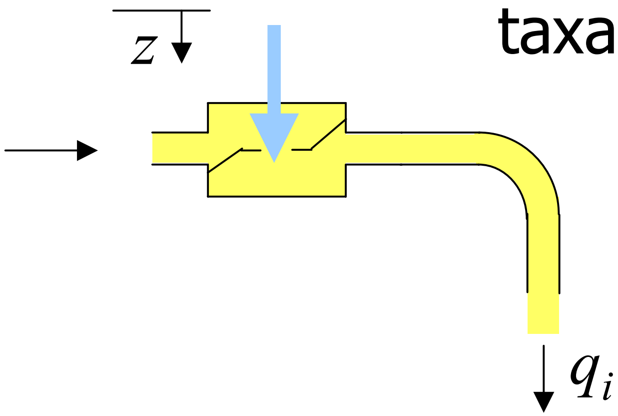
$$\frac{z}{h} = \frac{x}{y}$$

$$Z(s) = \frac{x}{y}H(s)$$



Válvula

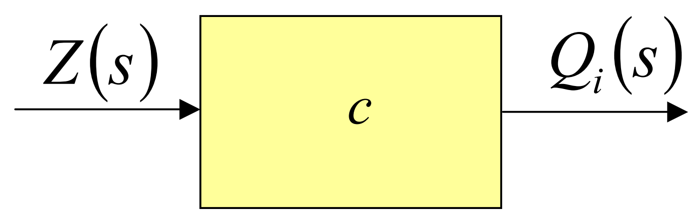
- A entrada z para a válvula determina a taxa de escoamento q_i na saída da válvula.



- A relação entre a entrada e a saída pode ser linearizada:

$$q_i = c\,z$$

$$Q_i(s) = c\,Z(s)$$

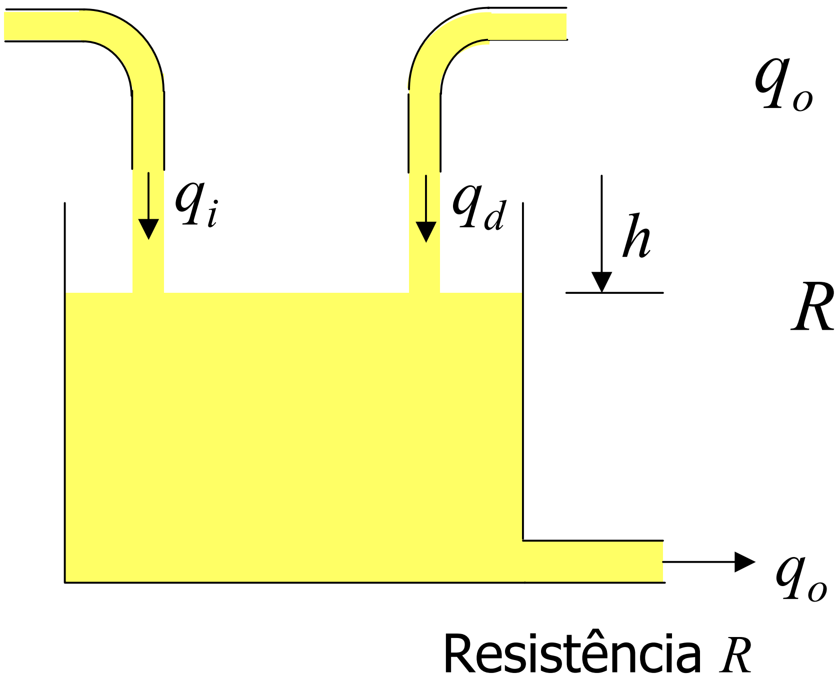


Tanque

Conservação da massa:

$$q_i + q_d - q_o = A\frac{dh}{dt}$$

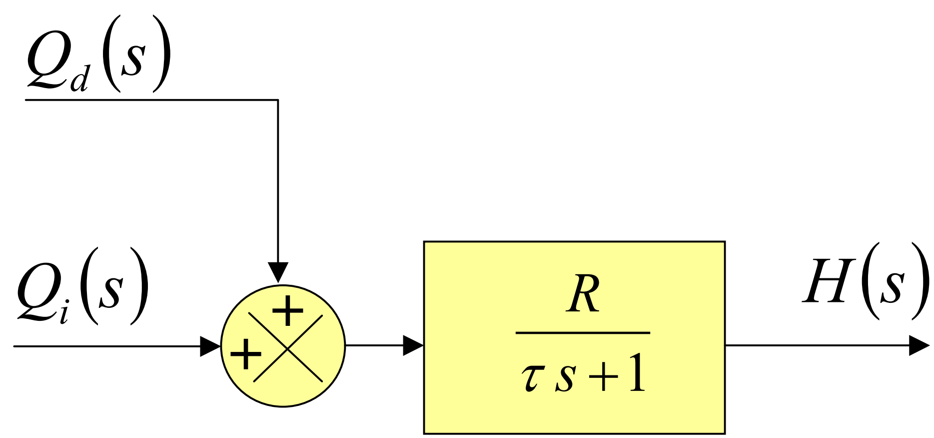
$$q_o = \frac{h}{R} \Rightarrow q_i + q_d = A\frac{dh}{dt} + \frac{h}{R}$$



$$R\left[Q_i(s) + Q_d(s)\right] = AR\,s\,H(s) + H(s)$$

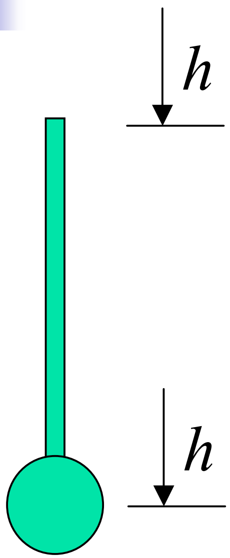
$$\tau = AR$$

$$H(s) = \frac{R\,Q_i(s)}{\tau s + 1} + \frac{R\,Q_q(s)}{\tau s + 1}$$



Bóia

A realimentação é o movimento da bóia



- O movimento da bóia transmite diretamente o sinal de altura de líquido em movimento da alavanca.

- Portanto a realimentação é unitária.

$$H(s) = 1.H(s)$$

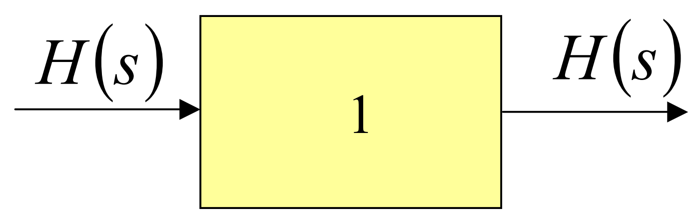
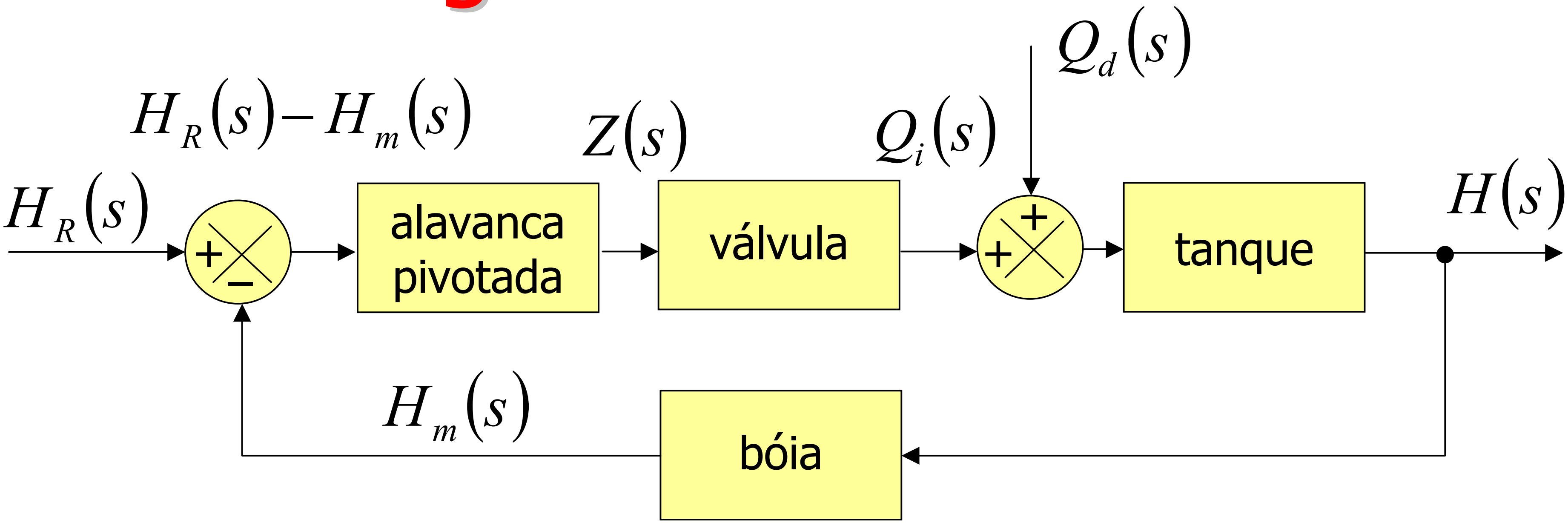
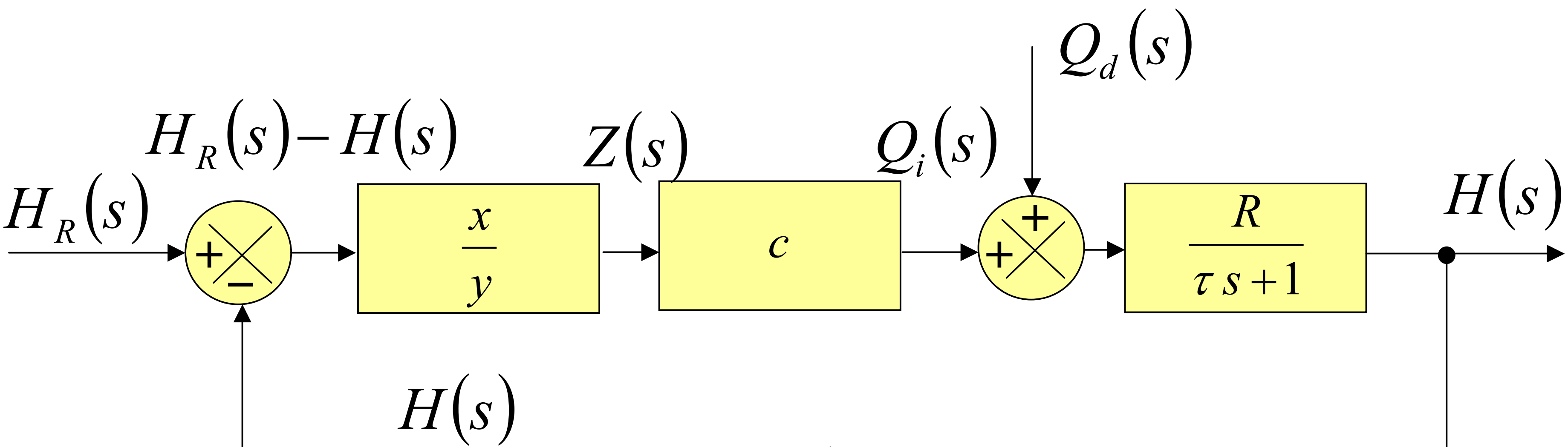
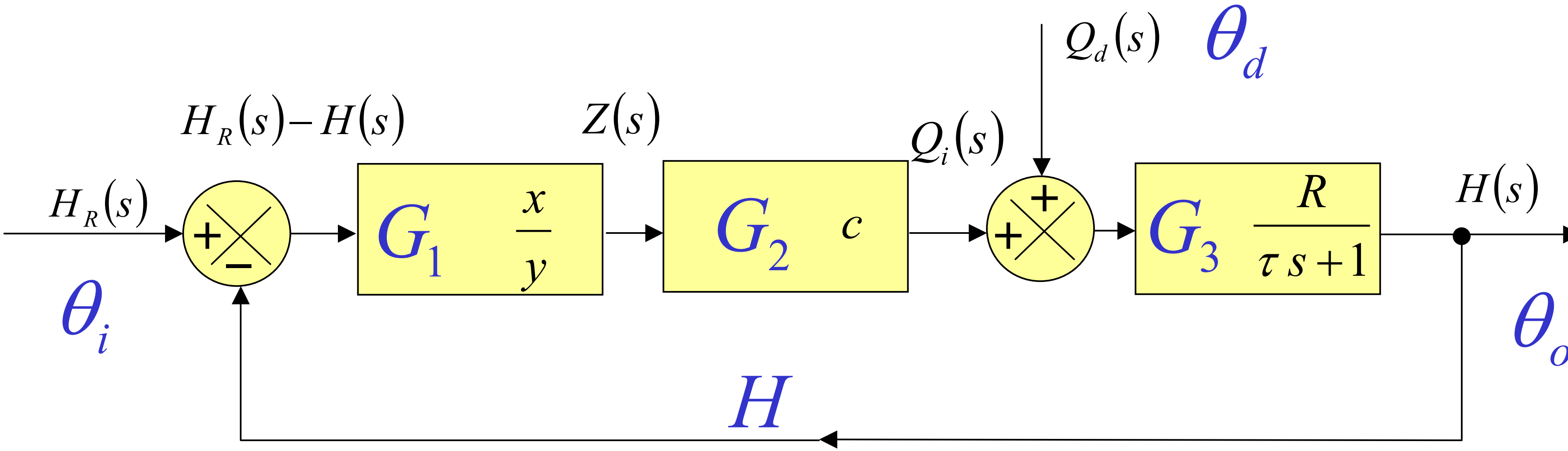


Diagrama de Blocos





Função de Transferência



$$\left[\left(\theta_i - \theta_o H\right) G_1\, G_2 + \theta_d\right] G_3 = \theta_o \qquad \theta_o = \frac{G_1 G_2 G_3\, \theta_i + G_3\, \theta_d}{1 + G_1 G_2 G_3 H}$$

$$H(s) = \frac{\dfrac{x}{y}\, c\, \dfrac{R}{\tau\, s + 1} H_R(s) + \dfrac{R}{\tau\, s + 1} Q_d(s)}{1 + \dfrac{x}{y}\, c\, \dfrac{R}{\tau\, s + 1}}$$

$$H(s)=\frac{\frac{x}{y}c\frac{R}{\tau s+1}H_R(s)+\frac{R}{\tau s+1}Q_d(s)}{1+\frac{x}{y}c\frac{R}{\tau s+1}} \Rightarrow H(s)=\frac{\frac{x}{y}cR\,H_R(s)+R\,Q_d(s)}{\tau s+1+\frac{x}{y}cR}$$

$$H(s)=\frac{\frac{x}{y}cR}{\tau s+1+\frac{x}{y}cR}H_R(s)+\frac{R}{\tau s+1+\frac{x}{y}cR}Q_d(s)$$

Definindo as constantes:

$$a=\frac{1+\frac{x}{y}cR}{\tau} \qquad K_1=\frac{\frac{x}{y}cR}{1+\frac{x}{y}cR} \qquad K_2=\frac{R}{1+\frac{x}{y}cR}$$

$$H(s)=\frac{K_1 a}{s+a}H_R(s)+\frac{K_2 a}{s+a}Q_d(s)$$

$$H(s)=\frac{K_1 a}{s+a}H_R(s)+\frac{K_2}{s+a}Q_d(s)$$

Com uma entrada degrau unitário em $H_R(s)$, sem distúrbio:

$$H_R(s)=\frac{1}{s} \qquad H(s)=\frac{K_1 a}{s(s+a)} \qquad h(t)=K_1\left(1-e^{-at}\right)$$

Com uma entrada degrau unitário em $Q_d(s)$, sem alterar o ponto de ajuste:

$$Q_d(s)=\frac{1}{s} \qquad H(s)=\frac{K_2 a}{s(s+a)} \qquad h(t)=K_2\left(1-e^{-at}\right)$$

Dados:

$$A = 0{,}3\,m^2 \qquad R = 300\,\frac{m}{m^3/s}$$

$$\tau = 90\,s$$

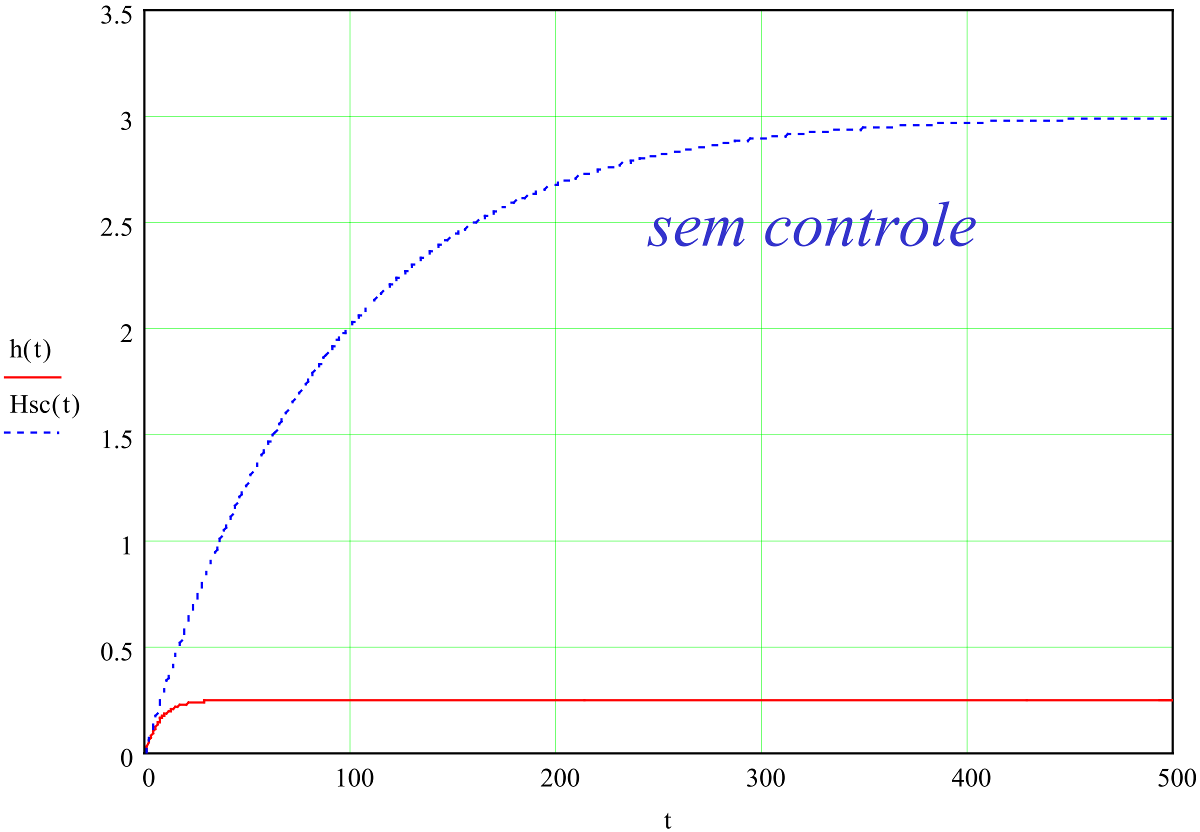
$$x = 0{,}3\,m$$

$$y = 1{,}2\,m$$

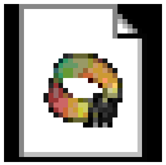
$$c = 0{,}15\,\frac{m^2}{s}$$

distúrbio $Q_d = 0{,}01\,\frac{m^3}{s}$

$$h(t) = K_2\left(1 - e^{-at}\right)$$



$$h(t) = 3\left(1 - e^{-0{,}011.t}\right)$$



Mathcad Document

$$h(t) = 0{,}25\left(1 - e^{-0{,}136.t}\right)$$